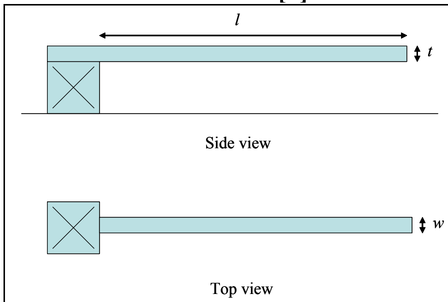
Quick and dirty MEMS design

Falco Systems Application note 3

Introduction
This document contains a number of equations to estimate the order of magnitude of forces, displacements and voltages of MEMS actuators. It can be used in “quick and dirty” MEMS design work. All the functional parts of one- and two-sides clamped beams, comb drives and parallel plate actuators are described. In all cases, bent beams (cantilever and clamped-clamped) are used to define the displacement for a certain force.
An excel sheet called quick_MEMS.xls belongs to this document. If the geometrical values are entered, it allows one to calculate voltages, forces, displacements and so on.
Note that these equations are very simple analytical models of rather complex electromechanical structures. They neglect anchoring point deformations, fringe fields, and so on. However, they should give the right order of magnitude for the variables involved, and can be used as a reality check for MEMS designs and as a complement to FE- (finite element) modeling.

Bent cantilever beams [1]



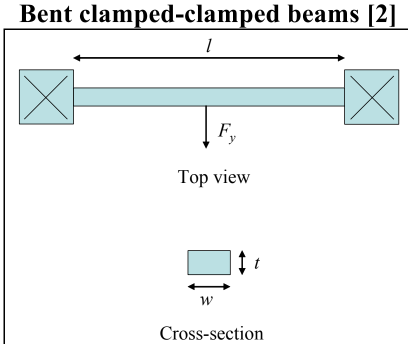
$$k_{spring} = \frac{3EI}{l^3}$$

$$I = \frac{tw^3}{12}$$

$$F = k_{spring}\delta$$

E = Young’s modulus
I = Moment of inertia
δ = deflection of the endpoint in the direction of w
F = applied force
k_{spring} = spring constant of bent beam

Bent clamped-clamped beams [2]



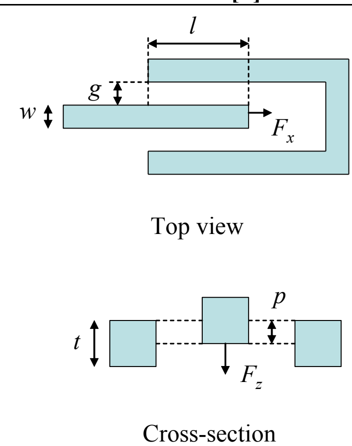
$$k_{spring} = \frac{24EI}{l^3}$$

$$I = \frac{tw^3}{12}$$

$$F = k_{spring}\delta$$

k_{spring} = the spring constant with a force acting on the middle of the beam
E = Young’s modulus
I = Moment of inertia
δ = deflection of the endpoint in the direction of w
F = applied force
Stretching of the beam is neglected.

Comb drive actuators [1]



$$F_x = \frac{\varepsilon p}{g} V^2$$

$$F_z = \frac{\varepsilon l}{g} V^2$$

$$F_{total} = nF$$

$$F = k_x \delta_x$$

ε = permittivity of free space
V = applied actuation voltage
n = number of comb fingers
δ_x = displacement of the comb drive in the x-direction
k_x = spring constant of supporting beam in x-direction

The “side instability voltage” limits the maximum usable actuation voltage of a MEMS comb drive. When the voltage becomes too high, the fingers of the comb drive can move sideward and touch the fingers at the other potential. This happens when the spring constant in the y-direction becomes less then the electrostatic force. The spring constant in the y-direction depends on the displacement in the x-direction (which in turn depends on k_x) and is given [3] as

$$k_y = \frac{200EI_{beam}}{3\delta_x^2 l_{beam}},$$

where I_{beam} is the moment of inertia as calculated in 2., and l_{beam} is the length of the beam. The sidewall instability occurs at the maximum displacement in the x-direction $\delta_x^{\max}$:

$$\delta_x^{\max} = -\frac{l_{beam}}{2} + \frac{1}{2}\sqrt{l_{beam}^2 + 2\frac{k_y}{k_x}g^2}\ .$$

Comb drives do not only move laterally, but generally also exhibit a significant out-of-plane motion, contrary to what one would expect from the figure. This happens because the electric field forcing a comb drive to move is not symmetrical (Fig. 1). It has the effect that the measured comb drive capacitance as a function of voltage will be influenced. This is the reason why, in the actuated case, usually $p \neq t$ as illustrated in the comb drive geometry figure. However, the levitation stops at a certain height when an equilibrium is found, which happens already at relatively low actuation voltages [4]. At higher actuation voltages, mostly the lateral deflection will change the capacitance.

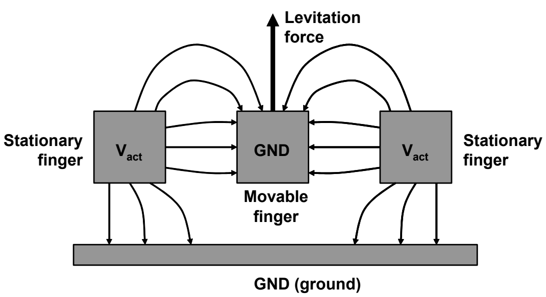
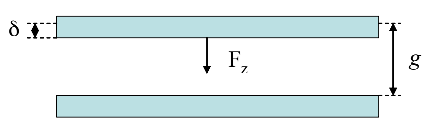


Figure 1. Cross-section of a comb drive. The field lines coming from the top cannot be counteracted by the field lines from below, because they are screened by the lower layer

Parallel plate capacitors [5]



$$F_z = \frac{\varepsilon A}{(g-\delta)^2}V^2 = -k_{spring}\delta$$

$$V_{pull-in} = \sqrt{\frac{8k_z g^3}{27\varepsilon A}}$$

$$V = \pm\sqrt{\frac{2k_z\delta(g-\delta)^2}{\varepsilon A}}$$

δ = deflection distance
g = equilibrium gap distance (no force)
A = capacitor area
k_{spring} = spring constant of bent beams (restoring spring)

The structure becomes unstable and collapses at the "pull-in voltage" $V_{pull-in}$, where $\delta = \frac{1}{3}g$.

High speed response of MEMS actuators [6

Cantilever beam resonance frequency:

$$f_1 = \frac{3.52}{2\pi\ l^2}\sqrt{\frac{EI}{\rho wt}}$$

Clamped-clamped beam resonance frequency:

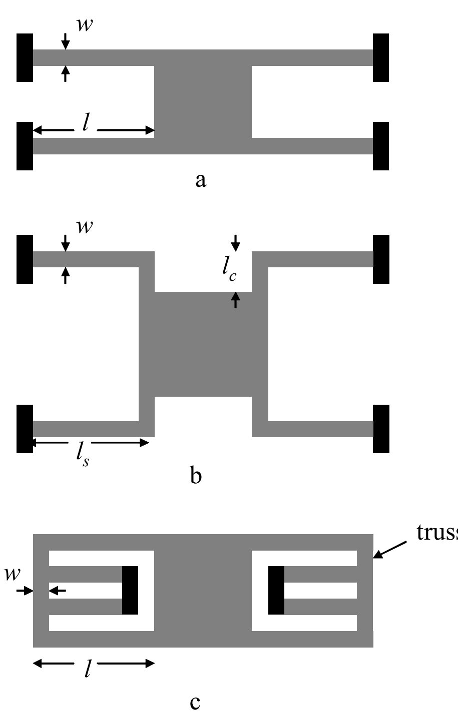
$$f_1 = \frac{22.4}{2\pi\ l^2}\sqrt{\frac{EI}{\rho wt}}$$

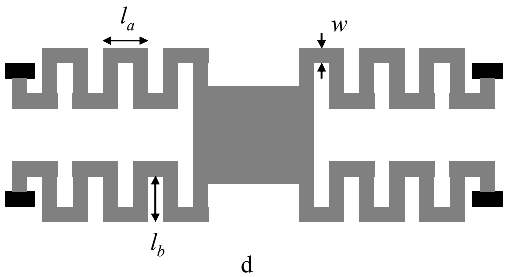
ρ = material density (kg/m^3)

In the case of the comb drive and the parallel plate capacitor, normally the mass of the supporting springs can be neglected. In that case we obtain a simple system with a single mass m (the mass of the comb drive or plate) and a single spring constant k (the sum of the spring constants of the beams). The resonance frequency is then:

$$\omega_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}\ .$$

Spring constant lowering structures [5]





d

a. Fixed-fixed flexure:

$$k = 4Ew\left(\frac{t}{l}\right)^3$$

b. Crab-leg flexure:

$$k = \frac{4Ew\left(\frac{t}{l_c}\right)^3}{1+\frac{l_s}{l_c}\left[\left(\frac{l_s}{l_c}\right)^2 + 12\frac{1+v}{1+\left(\frac{w}{t}\right)^2}\right]}$$

$$k \approx 4Ew\left(\frac{t}{l_c}\right)^3 \quad \text{for } (l_s >> l_c)$$

c. Folded flexure:

$$k \approx 2Ew\left(\frac{t}{l_c}\right)^3 \quad \text{for very stiff truss}$$

d. Serpentine flexure:

$$k \approx \frac{48GJ}{l_a^2\left(\frac{GJ}{EI_x}l_a + l_b\right)n^3}$$

$$\text{for } \quad n >> \frac{3l_b}{\frac{GJ}{EI_x}l_a + l_b}$$

with n is the number of meanders, $G = E/2(1+v)$ the torsion modulus, $I_x = wt^3/12$ the moment of inertia, and the torsion constant J given by

$$J = \frac{1}{3}t^3 w\left(1 - \frac{192}{\pi^5}\frac{t}{w}\sum_{t=1, i \text{ odd}}^{\infty}\frac{1}{t^5}\tanh\left(\frac{i\pi w}{2t}\right)\right)$$

In the specific case where $l_a >> l_b$, this reduces to

$$k \approx 4Ew\left(t/nl_a\right)^3.$$

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Falco Systems manufactures measurement electronics for MEMS research

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