



# Quick and dirty MEMS design

Falco Systems application note, version 2.0, www.falco-systems.com

W. Merlijn van Spengen, PhD vspengen@falco-systems.com

April 2018

#### Introduction

This equations summary form belongs to the comprehensive Falco Systems application note '*The electrostatic actuation of MEMS with high voltage amplifiers: from comb drive levitation and pull-in to dielectric charging and position noise*'. It lists the information required to do a fast reality check for MEMS designs, a 'quick and dirty' method to do a complete design or design assessment, and as a complement to FEM (finite element modeling). In addition, contrary to FEM, these equations give a lot of insight into why things are as they are. This document contains a number of equations to estimate the order of magnitude of forces, displacements and voltages of MEMS actuators. All the functional parts of one- and two-sides clamped beams, comb drives and parallel plate actuators are described. In all cases, bent beams (cantilever and clamped-clamped) are used to define the displacement for a certain force.

An Excel sheet called *quick\_MEMS.xls* also belongs to this document. Both the application note and the Excel sheet are available at <u>http://www.falco-systems.com/applications.html</u>. If the geometrical values are entered in the Excel sheet, it allows one to calculate actuation voltages, the corresponding forces, and the displacements.

### **MEMS** springs

An electrostatic MEMS actuator has two main properties: how the electric field exerts a force, and how the device is constrained by the spring by which it is suspended, and against which the electrostatic force acts. The magnitude of the displacement depends on the balance of this electrical driving force and the spring constant. The actual displacement  $\delta$  is given by the displacement distance where the electrical force F<sub>el</sub> and spring force F<sub>spring</sub> are equal and opposite in direction:

$$F_{el}(\delta) = -F_{spring}(\delta)$$

Table 1 contains a list of common springs with their analytical models, mostly taken from [13]. The displacement  $\delta$  to a force *F* is related by the spring constant of the structure *k*<sub>spring</sub>. The latter is determined by the geometry of the design and the material properties:

$$F = k_{spring}\delta$$

The symbols used are the vertical thickness *t*, the length *l* and width *w*, and variations on the length *l<sub>c</sub>*,  $l_a$  and  $l_b$  as indicated. The Young's modulus *E* is the material parameter describing the stiffness of the structural material itself, and used for the resonance frequency calculations are  $\rho$ , the density of the material in kg/m<sup>3</sup> and, for simplifying the equations, the moment of inertia

$$I = \frac{tw^3}{12}$$

When using analytical linear approximations for the spring constants it is important to check in which direction the equation is valid; the in-plane and out-of-plane spring constant are almost never the same. It should also be noted that important contributions to the spring constant are not taken into account, such as anchor point deformations. There also device-to-device and lot-to-lot variations in spring constant due to manufacturing process variations.

Table 1. MEMS spring equations





# **Electrostatic actuators**

The electrostatic force is required to calculate the displacement, given a certain spring constant. Table 2 contains the equations needed for an approximate calculation of the electrostatic force exerted by comb drive actuators and parallel plate actuators.

Comb drives do not only move laterally, but generally also exhibit a significant out-of-plane motion. This 'levitation' happens because the electric field forcing a comb drive to move is not symmetrical (Fig. 1). This effect is difficult to model analytically, so a FEM simulation is appropriate if more accuracy is required.



Figure 1. Cross-section of a comb drive. The field lines at the top side cannot be counteracted by the field lines from below, because the lines form below are screened by the ground plane





High speed response of MEMS actuators Cantilever beam resonance frequency

$$f_{res} = \frac{3.52}{2\pi l^2} \sqrt{\frac{EI}{\rho wt}}$$

Clamped-clamped beam resonance frequency

$$f_{res} = \frac{22.4}{2\pi l^2} \sqrt{\frac{EI}{\rho wt}}$$

If the mass of the springs is negligible compared to the total moving mass

$$f_{res} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

## **Disclaimer and copyright**

Although due care has been taken to ensure that the information contained in this application note is correct, Falco Systems does not assume any liability arising out of the application or use of any of the information described herein. Falco Systems reserves the right to make any changes to this application note without further notice. All text and images are © Falco Systems.

## References

[1] M. Elwenspoek, R. Wiegerink, Mechanical Microsensors, FSRM Training in Microsystems, FSRM, Neuchatel, 1999