

Driving piezoelectric actuators with high voltage amplifiers

PART II

The theory and practice of optimal electronic damping

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Introduction

Piezoelectric actuators ('piezos' in short) are often used in precision displacement systems. If driven correctly, they can offer atomic-scale position resolution. Because they have a high stiffness, they can also be used at high speeds. However, their high stiffness and low internal mechanical damping results in a tendency to severely resonate. Indeed, important commercial applications of piezos are their use in oscillators and watches, where their high resonance quality factor Q is used for high-stability single frequency generation. However, in piezo actuator positioning experiments in the laboratory this high Q is unwanted and means have to be found to lower it as much as possible to prevent displacement overshoot and prolonged ringing.

In the Falco Systems application note "Driving piezoelectric actuators with high voltage amplifiers, PART I - Piezo materials, applications, precision, speed, and damping of resonances" [1] a general overview of the use of piezos is given. In this application note PART II we discuss electronic damping of piezo actuators in depth. PART II is based on research carried out at Falco Systems regarding the exact nature of electronic piezo resonance damping. This application note serves as a practical introduction to the topic and offers enough information to understand why a damping resistor is necessary and to choose and use an optimum valued resistor. The interested reader who would like to have more background information is referred to the corresponding scientific paper by the same author [2] where full mathematical derivations and a comprehensive set of measurements are presented.

A piezo is usually actuated by a high voltage amplifier, which in this context is also called a 'piezo driver'. It will be shown that it is best not to connect a piezo directly to the piezo driver, but via a suitable coupling network that minimizes the ringing of the piezo. In most cases this coupling network can consist of a single, low-valued resistor. The inclusion of this resistor often changes the resonance amplitude and time it takes for the resonance to 'die out' by a factor ten or more. A number of Falco Systems high voltage amplifiers have an output series resistance already built-in with a value close to the optimal value for driving PZT (Lead Zirconate Titanate) based piezo actuators.

Other solutions to the damping problem such as adding mechanical (acoustic) damping (see application note part I [1]), low-pass filtering and transfer function/control-based input actuation signal shaping methods (see Appendix A) are also advantageous. These methods are complementary to the approach presented here. The better the intrinsic damping of a piezo system, the easier it is to implement these other methods.

Section 1 – Piezo resonance theory

The Butterworth – van Dyke model

A piezo has many resonances, of which the lowest, fundamental resonance usually is the strongest and the most cumbersome. The fact that the electrical and mechanical properties in a piezo are coupled makes the analysis somewhat complex. The electrical – mechanical analogy (Appendix B) can be used to understand how resonating voltages and currents in an electronic system are mathematically equivalent to force and velocity in a mechanical system.

The Butterworth – van Dyke equivalent schematic model of a piezo actuator (Fig. 1) was devised early in the 20th century to describe the fundamental resonance of a piezo [3]. It is a purely electrical model consisting of a parallel capacitance C_0 (the capacitance between the conducting plates of the

piezo) and a resonant ‘mechanical’ branch consisting of L_m , C_m and a damping resistor R_m . The subscript ‘m’ denotes that these components have a coupling to the mechanical domain. In fact, the current flowing through this mechanical branch is equivalent to the mechanical velocity of the piezo. The mechanical velocity of the piezo can in turn be integrated over time to calculate the piezo displacement.

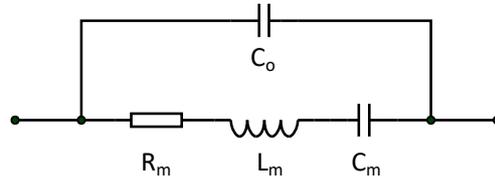


Figure 1. The Butterworth – van Dyke equivalent schematic of a piezo actuator

In the following sections we will see what happens to the resonant behavior of the Butterworth – van Dyke model when driven by a high voltage amplifier, either directly or via a suitable resistor.

Driving the piezo directly with a ‘hard’ voltage source

To intuitively see what happens when the piezo is driven by an ideal ‘hard’ voltage source (with no output resistance), we will initially assume that the internal damping of the piezo is zero too; that is $R_m = 0$ and $Q = \text{infinite}$ (Fig. 2).

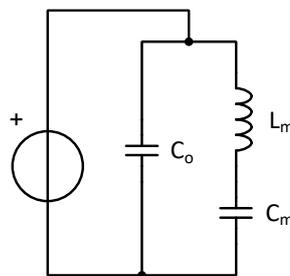


Figure 2. The no-loss piezo directly connected to a voltage source

The direct connection to the voltage source forces a constant voltage across the piezo at all frequencies and effectively decouples C_o from the resonant branch consisting of L_m and C_m . As a result, the current delivered by the voltage source which flows through the piezo is simply the current through C_o plus the current through L_m in series with C_m . As L_m and C_m constitute a series resonant circuit without damping, the current through this branch will go to infinity at its resonance frequency, called the ‘primary resonance’. To illustrate the theory, data of the Physik Instrumente P-820.20 piezo were used for creating the following graphs. This is the same piezo that was used for the experiments described in Section 3. In Fig. 3 and 4 the following curves are shown: the voltage across the piezo (fixed), the total current delivered by the voltage source to the piezo, the current flowing through the mechanical branch of the piezo (which is a measure for the piezo velocity) and the resulting piezo displacement curve (the integrated velocity).

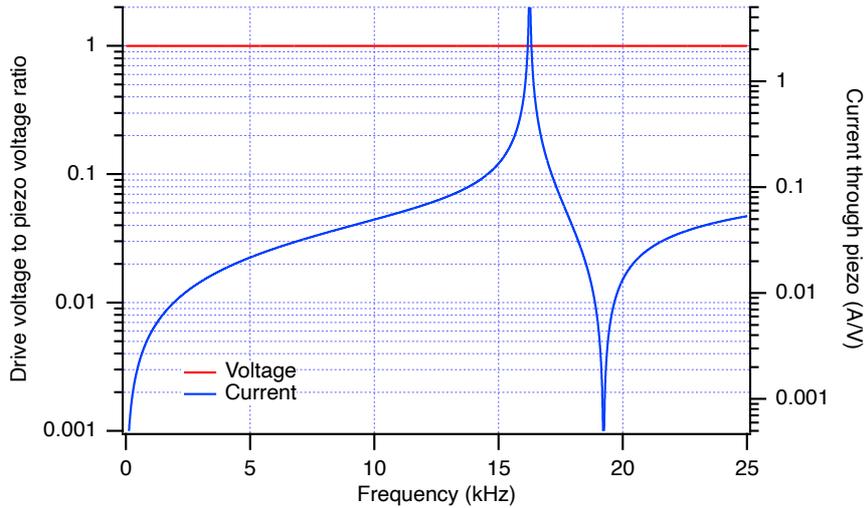


Figure 3. The primary resonance of the no-loss piezo when directly connected to a voltage source: voltage over and current through the piezo

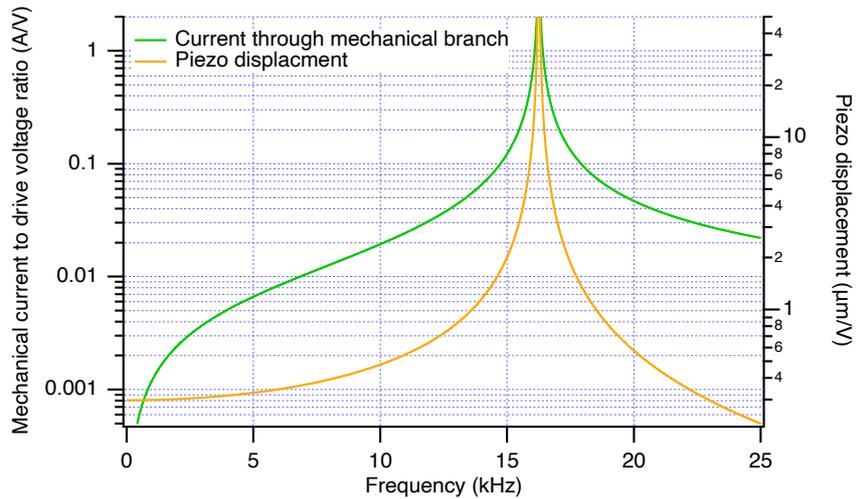


Figure 4. The primary resonance of the no-loss piezo when directly connected to a voltage source: current through the mechanical branch and displacement

We see that at low frequencies the current drawn by the piezo is low, and mainly caused by the need to charge and discharge C_0 . The impedance of the resonant mechanical branch is very high at low frequencies. The voltage to displacement ratio at low frequencies is constant: there is a single number, the piezo sensitivity, usually given in $\mu\text{m}/\text{V}$, that describes the displacement as function of the voltage. This sensitivity is given by the manufacturer as one of the specifications of the piezo. If the frequency is increased, we enter the range of frequencies where the resonant path becomes important. Exactly at the resonance frequency, the current through this path becomes infinitely high, as there is no damping in the model. A piezo will draw a very large current from the high voltage amplifier that is connected to it if driven at its resonance frequency! In return, the displacement of the piezo will go to infinity too. This is the reason why excessive overshoot and ringing is observed in the piezo displacement response when driven directly by a hard voltage source. To do better we have to add damping to the system.

Driving the piezo via a damping resistor

In Fig. 5 we see the same piezo, but now connected to the high voltage amplifier via a coupling resistor R_a . This resistor should be a high-power, low inductance type able to dissipate the heat and not influence the response by its own inductance. Wire-wound power resistors are unsuitable, but e.g. TO-220 thick or thin film resistors mounted on a heatsink are an excellent choice. With R_a present the

voltage source is no longer the only factor determining the voltage across the piezo. Now C_o is coupled to the mechanical branch and will influence the resonance frequency.

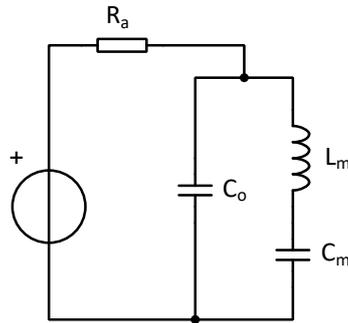


Figure 5. The no-loss piezo connected to a voltage source via a resistor R_a

In Fig. 6 we see the voltage across the piezo for different values of R_a . With $R_a = 0$, we regain the situation we have seen before. But if R_a is not zero, the voltage across the piezo drops at higher frequencies because R_a forms an RC low pass filter with C_o . In addition, the voltage across the piezo will go to zero at the primary resonance as the impedance of the mechanical branch is zero at this frequency. But a third effect is visible in these curves too: there is another resonance at a higher frequency than the primary resonance. This is the ‘anti-resonance’, where C_m , L_m and C_o all play a role. Contrary to the primary resonance, the anti-resonance is a parallel resonant circuit. In a parallel resonant circuit the impedance goes to infinity at the resonance frequency; at this frequency, the piezo draws essentially no current.

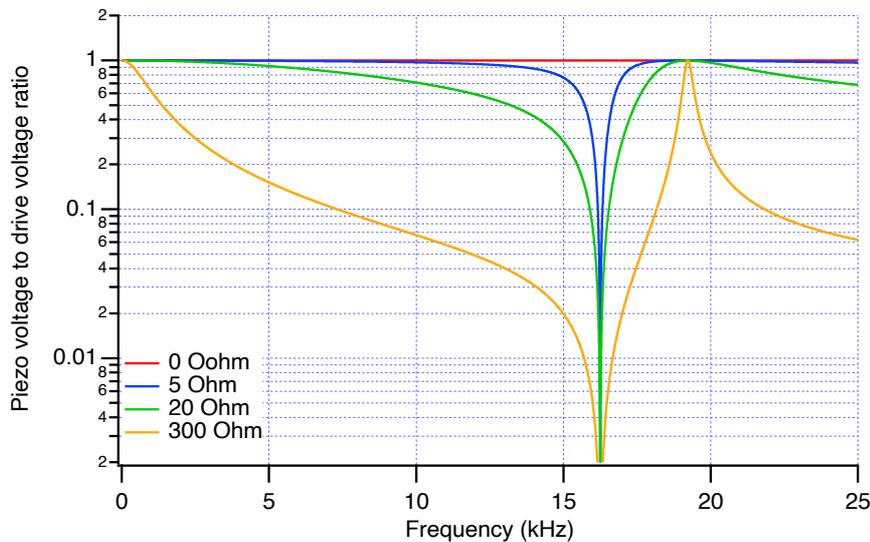


Figure 6. Voltage across the no-loss piezo

The current delivered by the amplifier as drawn by the piezo (Fig. 7) is again a combination of the (now RC low-pass filtered) current required to charge C_o at low frequencies and the current defined by the resonances at high frequencies. As the actual voltage across the piezo drops at the primary resonance, the current drawn by the piezo is not infinite anymore. With increasing R_a the resonance becomes completely negligible. The current is zero at the anti-resonance frequency, irrespective of the exact value of R_a .

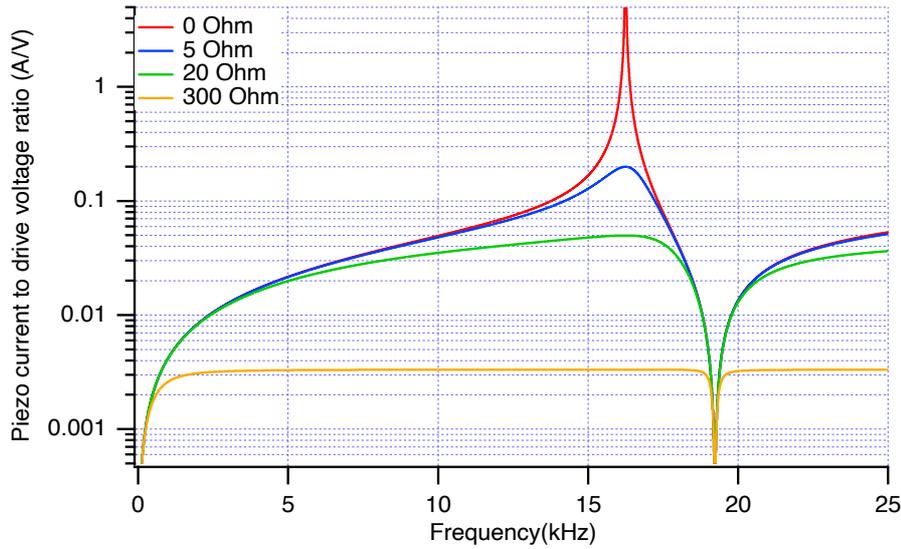


Figure 7. Current through the no-loss piezo

Although the piezo draws hardly any current from the high voltage amplifier at the anti-resonance frequency, the resonant current circling back and forth between C_o , C_m and L_m is large at this frequency, depending on the value of R_a . As a result, the current through the mechanical branch consisting of L_m and C_m is large (Fig. 8).

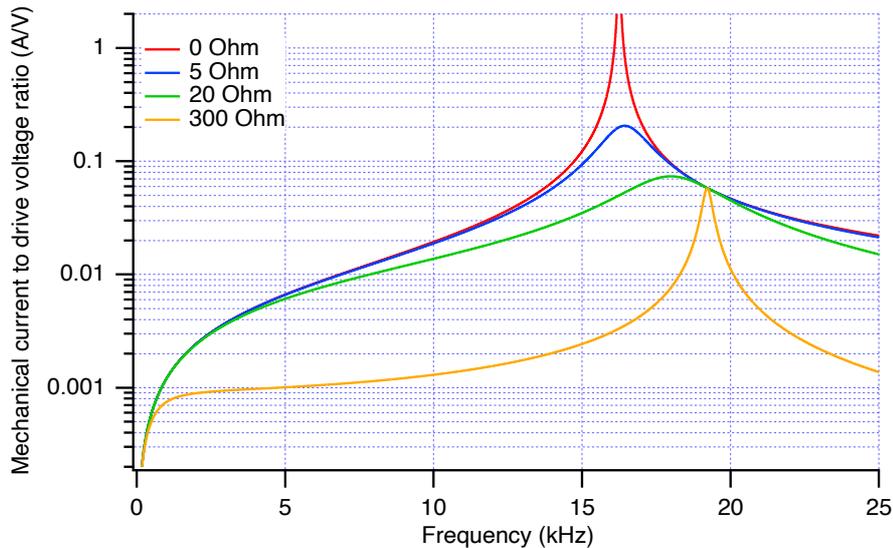


Figure 8. Current through the mechanical branch of the no-loss piezo and corresponding displacement transfer functions

The integrated current through the mechanical branch of the piezo model is a measure for the piezo velocity. By integration, we again obtain the corresponding displacement transfer curves (Fig. 9). Here we clearly see the beneficial effect of adding R_a . For low values of R_a the primary resonance is strong. When R_a is increased, the resonance frequency begins to shift, and the quality factor Q of the resonance drops: the peak becomes broader and lower than the undamped curve. If we increase the value of R_a even more, the resonant response becomes sharp again, and is now located at the anti-resonance frequency. The amplitude of the resonance is much lower now, because the voltage of the high voltage amplifier is low-pass filtered by the RC filter consisting of R_a and the piezo capacitance. When too high a value of R_a is used, a slow piezo displacement response will be observed with a small amplitude but long ringing time on top.

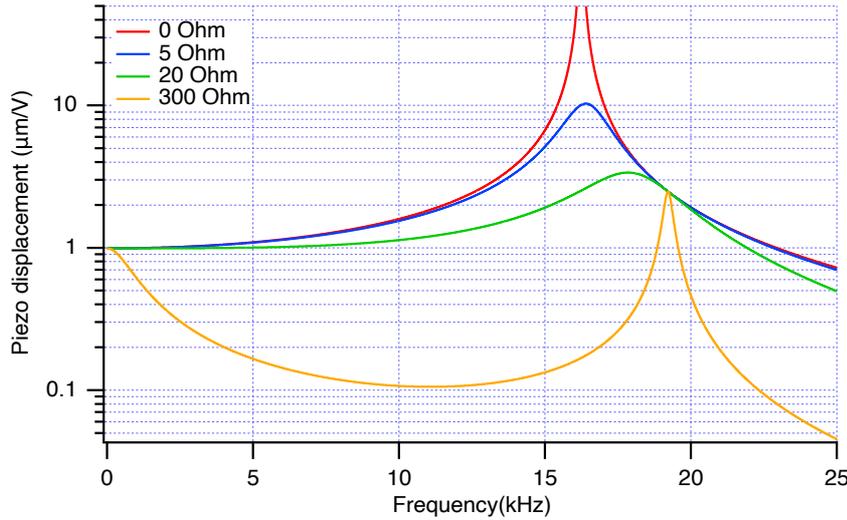


Figure 9. Corresponding displacement transfer functions

Section 2 – The theory of optimal damping

The analysis of the piezo behavior presented above clearly suggests that there is an ‘optimum’ value of R_a . It should not be so low that the primary resonance is still large. But it should also not be so high that the piezo response becomes too much low-pass filtered and very slow, with a sharp resonance present at the anti-resonance frequency. The optimal value of R_a shifts the resonance halfway in between the primary resonance and anti-resonance frequencies, hence we will call it $R_{a,mid}$. The value of $R_{a,mid}$ can be calculated and turns out to be (see the calculations presented in [2])

$$R_{a,mid} = 2 \sqrt{\frac{C_m L_m}{C_o (4C_o + C_m)}}$$

The resonance cannot usually be completely suppressed with this value of resistance, but the quality factor Q of the resonance will be minimized. $R_{a,mid}$ can be determined if C_o , C_m and L_m are known. These model parameters can in principle be determined by curve-fitting a measured transfer function curve of the piezo current to drive voltage ratio, but this is a rather cumbersome process.

Fortunately, it turns out that the ratios of C_o , C_m and L_m are more or less the same for all PZT (Lead Zirconate Titanate) based piezo actuators. Hence the optimal coupling resistor value $R_{a,mid}$ is the same for all such piezos, and its value is around 20Ω . Due to piezo-to-piezo variation and factors such as temperature dependency of these parameters, the voltage dependence of C_o and ageing, it is advisable to build in some margin on this number for actual use. An R_a value that is a little too small will immediately result in a high amplitude primary resonance. A value of R_a that is a little too large will merely slow the system down somewhat and cause a small increase in the time it takes for the low-amplitude ringing to ‘die out’. Therefore a resistance value slightly larger than $R_{a,mid}$ calculated above is advisable. Falco Systems recommends the use of a 50Ω output damping series resistor when driving PZT piezos. A number of Falco Systems high voltage amplifiers have this ‘ideal’ 50Ω output series resistor already built-in. When used as piezo drivers these amplifiers offer the optimal solution for driving PZT piezo actuators with minimal resonances and do not require any external components.

Sometimes the use of a separate inductor L_a in series with R_a is advocated. When carefully implemented the piezo resonance can be suppressed somewhat better than with R_a alone, but the use of such an inductor cannot usually be recommended. The values of C_o , C_m and L_m need to be accurately known, and the exact values of the damping components R_a and L_a are critical. If the values are not exactly right, the resonance amplitudes will be higher rather than lower compared to using R_a alone. For more details, see again [2].

A further possibility is to adjust the value of C_o . An extra capacitor connected across C_o will bring the primary resonance and anti-resonance frequencies closer together, which makes the resonance peak at $R_{a,mid}$ sharper. This is the opposite of the effect that we are after. We would need a negative capacitance across C_o to lower the quality factor of the resonance. This can in fact be accomplished by a clever compensation scheme requiring three high voltage amplifiers but is not easily implemented (see Appendix C).

Section 3 – The piezo with internal loss and experimental examples

Mechanical damping

Piezos will not behave exactly as described in Section 2, because they have some internal mechanical damping, modeled by R_m in the Butterworth – van Dyke model of Fig. 1. The effect of this resistor is that it reduces the Q of the primary resonance and the anti-resonance; the resonance quality factor is no longer infinite. If R_m is taken into account, an almost exact fit to actual piezo behavior is obtained for all features that we have discussed so far: voltage across the piezo, current through the piezo, current through the mechanical branch of the piezo model (and hence the piezo velocity) and the mechanical displacement of the piezo.

The inclusion of R_m creates many new terms in the algebra describing the transfer functions of the system. The complete functions are given in [2], and were used for fitting the experimental data shown below. But it will usually be easier to simulate a SPICE model of the network instead.

The main effect of including R_m is that the resonances become less sharp already in the absence of coupling resistor R_a . For the piezo used in the experiments the unloaded quality factor of the primary resonance was $Q \approx 20$ instead of infinity, which is perfectly modeled by the inclusion of an R_m with a value of a few Ohms.

Measuring the piezo properties

To illustrate the theory presented above a small optical table was built with a piezo displacement measurement system mounted on it. An interferometer was built on the optical table to be able to measure the small piezo displacements using a laser, a beam splitter cube and a photodetector and a small reflector glued onto the piezo itself (Fig. 10). The optical table was small to prevent its own resonances from influencing the results (see the explanation of the mechanical loop, application note PART I [1]). The first unsatisfactory experiments were performed with optical tables made of aluminum and granite, but these added too many resonances of their own in the frequency range of interest. In the end, a 10 x 6 x 4 cm solid block of lead (a very heavy and highly damping material) was used, on rubber feet.

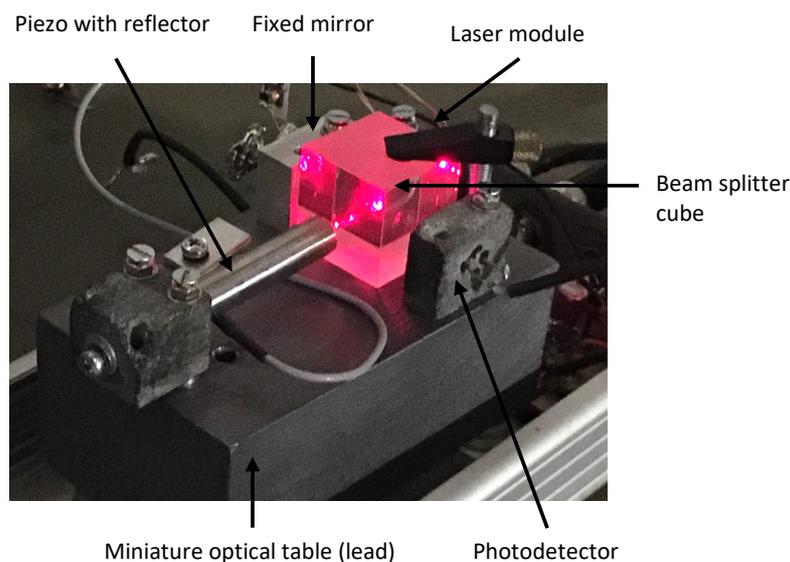


Figure 10. The optical table with the piezo and the interferometer

The piezo was a 10N preloaded Physik Instrumente P-820.20 model. 'Preloaded' means that the piezo enclosure has a built-in spring so that a compressive force is added to the piezo at all times. This is important because a piezo will easily crack under tensile forces but not under compressive forces. The frequency sweeps of this experiment will severely stress the piezo at high frequencies both under elongation and compression due to the resonances present. Do not even think of performing such a frequency sweep experiment on an unloaded piezo. The author did try it, and it is not a good idea. The force on the piezo is simply $F = ma$ (force = mass times acceleration). The higher the frequency, the higher the acceleration and hence the higher the force that can stress the piezo to the breaking point. Even very low excitation voltages such as the $2.5\text{mV}_{\text{rms}}$ used in the frequency sweep experiment shown here will result in high forces at high frequencies, because piezos are mechanically so fast.

Frequency domain measurements

The fact that the inclusion of R_a is indeed very beneficial is clearly visible in the experimental results shown in Fig. 11. Note that this is a log scale and that the primary resonance is damped by a factor 10 when the proper value of $R_{a,\text{mid}} = 22\Omega$ is included. It is also shifted halfway in between the primary resonance and anti-resonance frequencies, as predicted. However, it is impossible (for this piezo, with its own particular value of internal damping R_m) to get rid of the resonance entirely. The lowest Q that can be obtained depends on the internal damping properties of the particular piezo used in the experiment.

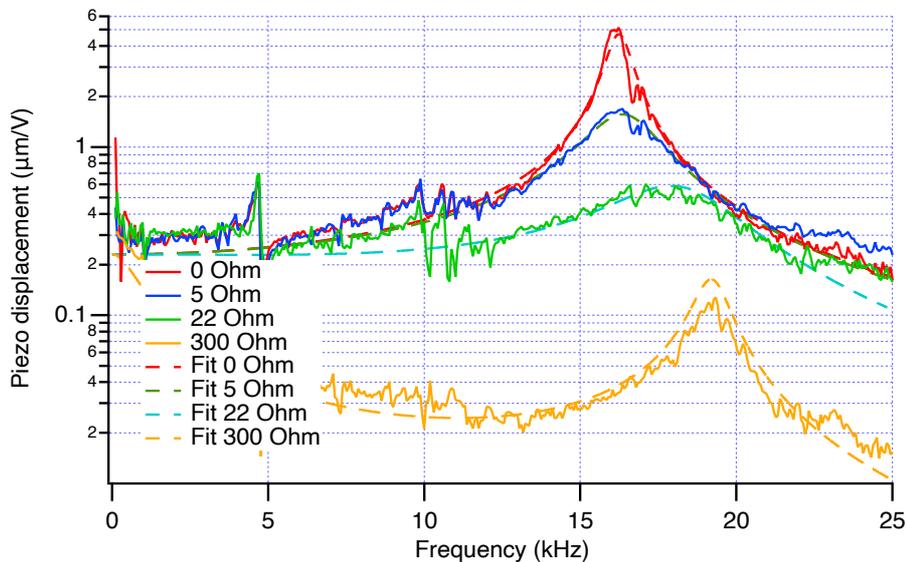


Figure 11. Measured piezo displacement transfer functions for different R_a and theoretical fit

Time domain measurements

A resonance peak in the frequency domain results in overshoot and ringing in the step response in the time domain. In Fig. 12 the measured step response of the piezo displacement is shown for different values of R_a . With no external damping the overshoot is high, and the ringing takes a long time to die out, even longer than the period of the square wave actuation used to drive the piezo in this experiment. As expected, the response is most damped and fairly similar for $R_a = 22\Omega$ and 50Ω , with 50Ω being the safer value far away from the primary resonance. In the $R_a = 300\Omega$ experiment, a low pass filtered, low amplitude, but persistent resonant wiggle is visible close to the anti-resonance frequency. More details and experimental results of the frequency and time domain measurements are given in [1].

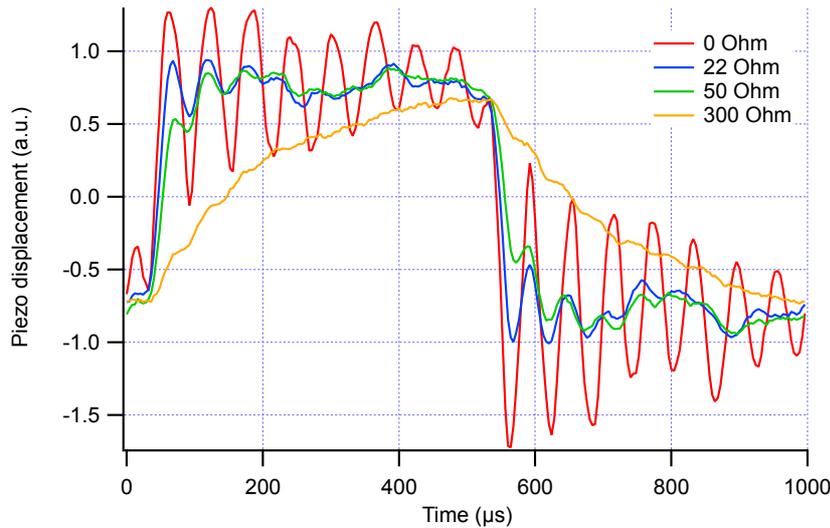


Figure 12. Step response of the piezo displacement for different values of R_a

Section 4 – Conclusions and recommendations

The piezo with its first (lowest) mechanical resonance can be modeled as an electronic circuit as given in Fig. 1. The mechanical response is equivalent to the integral of the current over time through the mechanical branch, defined by L_m , C_m , and R_m . C_o is the electrical capacitance as measured at low frequencies between the terminals of the piezo. When the piezo is driven with a ‘hard’, low output resistance piezo driver, a large mechanical resonance (the ‘primary resonance’) is present in the response. This resonance is the series resonance caused by C_m and L_m . Its quality factor is determined by R_m . It cannot be seen by monitoring the voltage across the piezo, but the current drawn from the piezo driver becomes large at this frequency.

If we place a large value resistor R_a in series with the piezo (Fig. 13), the bandwidth of the system is limited by the RC filter formed by this resistance R_a and the piezo capacitance C_o . Instead of the primary resonance, there will be another mechanical resonance visible (the ‘anti-resonance’) that is at a slightly higher frequency than the primary resonance. It is a parallel resonance which includes C_o . This resonance can be sharp (it has a high quality factor) but its absolute amplitude is lower than the primary resonance because of the low-pass filter effect. The larger R_a , the sharper this resonance becomes.

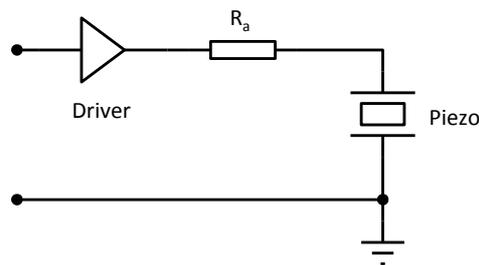


Figure 13. Piezo connected via a series resistance R_a

The value of a ‘midpoint’ series resistance $R_{a,mid}$ can be calculated where the quality factor of the resonance is lowest. In this case the resonance frequency is halfway in between the primary resonance and the anti-resonance frequencies. The quality factor of the resonance is lowest at $R_{a,mid}$, which is around 20 Ohm for PZT-based piezo’s. The absolute magnitude of the resonance is lower still for slightly higher R_a values, where the resonance frequency approaches that of the anti-resonance, but the quality factor is still quite low. It is important to have some margin on R_a because of individual differences between piezo types and the significant changes of C_o with temperature and

actuation voltage. For most piezo's the corresponding recommended value is around 50Ω , which a number of Falco Systems high voltage amplifiers have already built-in. A large current may flow through R_a . If R_a is implemented as a stand-alone resistor (as opposed to the internal resistor of the Falco Systems amplifiers), make sure that it is an appropriate high-power one. Because of the high frequencies involved, a low-inductance thick film resistor is better than a wire-wound resistor.

More advanced compensation schemes have been described in the literature, notably the inclusion of an external inductor in series with R_a to create a series resonant circuit and short-circuit the remaining resonance peak (RL-damping). The disadvantage is that only one value of inductor and one value for R_a can be used: there is no room in this compensation scheme to employ the RC filter effect. Also, the circuit is much more sensitive to variations in the exact values of the components. Especially the variations of C_o with actuation voltage and temperature are so large that a passive RL damping is less effective than would be expected based on theory alone. Another possibility for extra compensation is to reduce the effect of C_o by the inclusion of a negative capacitance in parallel with the piezo. Because passive negative capacitances do not exist, this approach requires an 'active' implementation with one or more extra high voltage amplifiers. The added complexity and potential instability limit the usefulness of this approach for most applications.

The final 'quick fix' recommendation is to at least include a $R_a = 50\Omega$ resistor in series with the piezo. This solution will lower the piezo mechanical resonance significantly and is robust against piezo variability. If the lower resonance peak near the anti-resonance frequency is still troublesome, it can be counteracted further by processing the input signal sent to the piezo driver with e.g. a computer implementation of the inverse (electrical to mechanical!) transfer function. This inverse transfer function approach works best in practice if a significant series resistance R_a is present to lower the sharpness of the resonance peak first.

References

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Appendix A – Model-based signal shaping

If the resulting peak in the mechanical piezo response is still a problem after compensation with $R_{a,mid}$, further optimization can be achieved by modifying the driving voltage with a model of the inverse of the electrical-to-mechanical piezo-plus-driver transfer function. The inverse transfer function can be obtained in several ways, but the primary distinction is between using a physical model and using the direct inverse of a (smoothed) measured response. Important is that the inverse transfer function cannot be extended to infinitely high frequencies. If we would do so, its amplitude would have to go to infinity due to the fact that the piezo response itself goes to zero. Therefore, we have to choose a cut-off frequency, the highest frequency for which we want to inverse transfer function to operate. A natural choice can be either the frequency of the first resonance of the piezo, or the frequency defined by the RC filter caused by R_a and C_o . But higher compensation frequencies for extended bandwidth are also possible. A good example of this approach is the signal shaping of the drive signal of the piezo in an AFM (atomic force microscope) measurement setup [4].

Appendix B – The electrical - mechanical analogy

Mechanical mass-spring-damper systems behave mathematically the same as electronic circuits with inductors, capacitors and resistors. Therefore mechanical-electrical analogies have been devised, notably the impedance analogy and the mobility analogy. In the impedance analogy, electrical voltage is replaced by force, current by velocity, and so on, see Table I. In particular, the inductance, capacitance and resistance of a circuit have a direct mechanical analog in the mass, spring and damper of a mechanical system.

Table I – Elements of the mechanical-electrical analogy [5]

	Electrical domain	Mechanical domain
Voltage $v \rightarrow$ Force F	v	F
Current $i \rightarrow$ Velocity u induced by the force	i	u
Resistance $R \rightarrow$ Mechanical resistance/damping c	R	c
Ohm's law	$v = i \cdot R$	$F = u \cdot c$
Impedance \rightarrow Mechanical impedance	Z	Z_{mech}
Generalized Ohm's law	$v = i \cdot Z$	$F = u \cdot Z_{mech}$
Inductance $L \rightarrow$ Mass m	$v = L \frac{di}{dt}$	$F = m \frac{du}{dt}$
Impedance of $L \rightarrow$ Impedance of m	$j\omega L$	$j\omega m$
Capacitance $C \rightarrow$ Compliance $C_s = 1/k$	$v = \frac{1}{C} \int i \cdot dt$	$F = \frac{1}{C_s} \int u \cdot dt$
Impedance of $C \rightarrow$ Impedance of $1/k$	$\frac{1}{j\omega C}$	$\frac{1}{j\omega C_s}$

With spring constant k , time t , angular frequency ω , and $j^2 = -1$

In normal mechanical systems, these equivalences are useful e.g. to design mechanical transfer functions using electrical filter theory, but they are no more than an analogy. In a purely mechanical system, there are no current, no voltage, but only velocity and force. In an electromechanical system, such as a piezo or MEMS device, where the mechanical response directly affects the electrical properties of the system and vice versa, the analogy leads to a direct equivalence of the mechanical properties and their electrical twins. In this case, the mechanical response can be modeled as an electrical circuit that behaves as if it were really an electronic circuit. Here, the 'analogy' is not just an analogy, but a real system property. Table II lists the information that is needed to create an electronic equivalent circuit of a simple second order mechanical mass-spring damper system. In Fig. 14 this transformation from the mechanical to the electrical domain is shown schematically.

Table II – The transformation of electrically equivalents of electromechanical elements

	Mechanical element	Electrical equivalent	Transformation
Mechanical resistance	c	R_m	Ns/m \leftrightarrow Ω
Mechanical inductance	m	L_m	kg \leftrightarrow H
Mechanical capacitance	C_s	C_m	m/N \leftrightarrow F

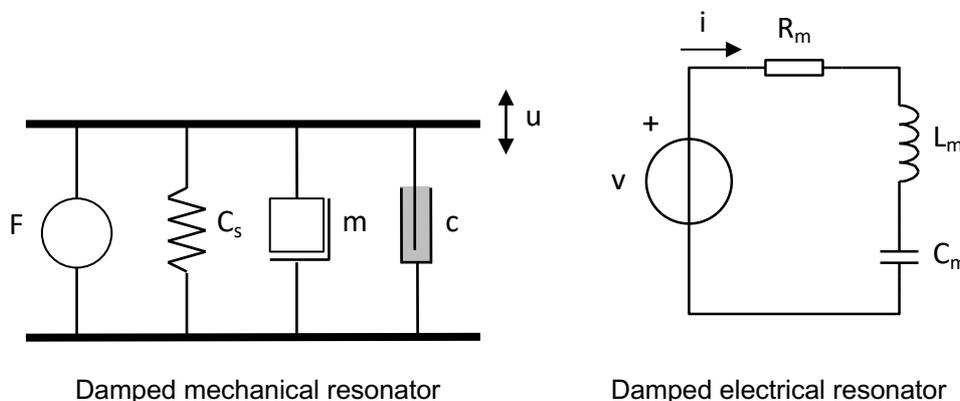


Figure 14. The mechanical - electrical transformation

A good example of such a coupled electrical – mechanical model is the Butterworth – van Dyke model of Fig. 1. The electrical capacitance that can be measured between the piezo terminals at low frequencies is modeled as C_o . The mechanical behavior enters the model via the resonant circuit of C_m , L_m and R_m . Due to the fact that these components are coupled to C_o , the model is not a standard second order system.

The transformation works both ways: if we know the electrical circuit parameters, we can evaluate the mechanical response. The force F and velocity u of the piezo are the mechanical response to a driving voltage v and a current i . Van Dyke [3] has shown that the current through the mechanical branch consisting of C_m , L_m and R_m is equivalent to the velocity u of the piezo. As displacement is the integral over time of velocity, $d = \int u dt$, in this way the piezo displacement amplitude d can be evaluated this way.

Appendix C – Electronic reduction of C_o

There is an important difference between the primary resonance of the low R_a and the anti-resonance of the high R_a case. In the latter we have direct voltage access to the resonant node: it is a parallel resonance. In the mechanical series resonance, the resonant node is in between C_m and L_m , and inaccessible; it is not even electrical. For the anti-resonance, the node is at the connection of C_o and the mechanical piezo branch. If we connect an extra capacitor C_a across the piezo, effectively enlarging C_o , we can shift the anti-resonance frequency electrically: it comes closer to the primary resonance frequency with increasing C_a , and this effect is used in ‘tuning’ electronic crystal oscillators. Such an electrical influence on the primary resonance frequency is not possible.

If we can lay the anti-resonance close to the primary resonance by increasing C_o by placing a C_a in parallel with it, we should be able to shift the anti-resonance away from the primary resonance by reducing C_o . This can be done by placing a negative capacitor over C_o . This effectively lowers the quality factor of the piezo resonances.

Unfortunately, passive negative capacitors do not exist. Active circuits that mimic the behavior of a negative capacitor can be constructed by using a negative impedance converter (NIC) circuit. These come in two varieties, the current-inverting negative impedance converter (INIC) and the voltage-inverting negative impedance converter (VNIC), Fig. 15.

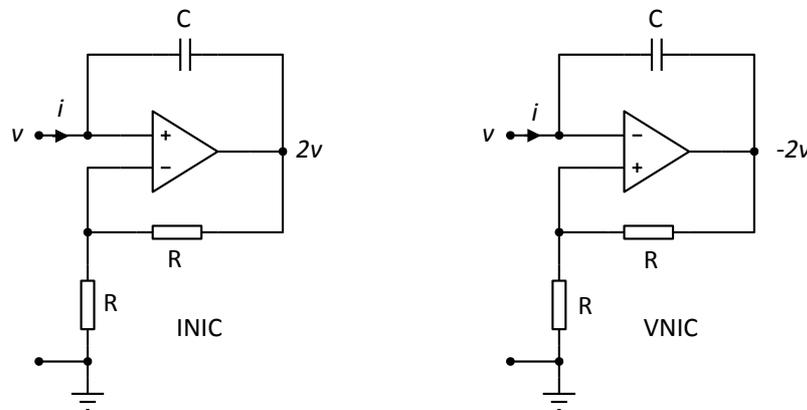


Figure 15. Circuits for simulating a negative capacitance

The INIC is unstable at high input impedances and the VNIC at low input impedances, because in both cases there should be more negative feedback in the circuit than positive feedback for the NIC to be stable. Although some success has been achieved in lowering the piezo resonance peak this way [6], the fundamental problem with this approach is that a piezo has a widely varying impedance as a function of frequency. This causes any NIC to be unstable at some frequencies, and hampers the implementation of such a solution. A possibility may be to use two NIC's, but this results in a requirement for three high voltage amplifiers instead of one, two of which are required to simulate the negative capacitances.